13. Präsenzübung zur Quantentheorie II, SS 2001

(zu bearbeiten am Donnerstag, 5.7.2001)

Aufgabe P21 *Wicksches Theorem für skalares Feld*

Das quantisierte skalare Feld lautet

$$\phi(x) = \int d\tilde{k}\,[a(\vec{k})e^{-ikx} + a^\dagger(\vec{k})e^{ikx}] =: \phi^+(x) + \phi^-(x)\ .$$

Zeigen Sie explizit (mit der Notation $\phi(i) := \phi(x_i)$):

a)

$$\begin{aligned} \phi(1)\phi(2) &= :\phi(1)\phi(2): + [\phi^+(1), \phi^-(2)] \\ T\phi(1)\phi(2) &= :\phi(1)\phi(2): + \langle 0|\, T\phi(1)\phi(2)\, |0\rangle \end{aligned}$$

b)

$$\begin{aligned} \phi(1)\phi(2)\phi(3) &= :\phi(1)\phi(2)\phi(3): + [\phi^+(1), \phi^-(2)]\,\phi(3) \\ &\quad + [\phi^+(2), \phi^-(3)]\,\phi(1) + [\phi^+(1), \phi^-(3)]\,\phi(2) \\ T\phi(1)\phi(2)\phi(3) &= \ldots \end{aligned}$$

c)

$$T : \phi(1)\phi(2) :\, \phi(3) = \ldots$$

d)

$$\begin{aligned} \phi(1)\phi(2)\phi(3)\phi(4) &= :\phi(1)\phi(2)\phi(3)\phi(4): + :\phi(1)\phi(2): [\phi^+(3), \phi^-(4)] \\ &\quad + :\phi(1)\phi(3): [\phi^+(2), \phi^-(4)] \\ &\quad + :\phi(2)\phi(3): [\phi^+(1), \phi^-(4)] \\ &\quad + [\phi^+(1), \phi^-(2)]\,(:\phi(3)\phi(4): + [\phi^+(3), \phi^-(4)]) \\ &\quad + [\phi^+(2), \phi^-(3)]\,(:\phi(1)\phi(4): + [\phi^+(1), \phi^-(4)]) \\ &\quad + [\phi^+(1), \phi^-(3)]\,(:\phi(2)\phi(4): + [\phi^+(2), \phi^-(4)]) \\ T\phi(1)\phi(2)\phi(3)\phi(4) &= \ldots \end{aligned}$$

e) Was ist die graphische Darstellung von $\langle 0|\, T\phi(1)\phi(2)\phi(3) : \phi^3(4) : |0\rangle$, wobei $\langle 0|\, T\phi(1)\phi(2)\, |0\rangle =: 1 \bullet\!\!-\!\!\!-\!\!\bullet\ 2$?

Anmerkung: Nützliche Abkürzungen sind $(12) := \theta(t_1 - t_2)$ und $[\phi^+(1), \phi^-(2)]\,\phi(3) =: [1,2]\,3$.